Online Appendix

A Mortgage contracts: Math and notation

Fixed-rate mortgages

Fixed rate mortgages are mortgages with a constant interest rate r, and a constant per-period payment determined such that the mortgage is fully amortized over T periods. Let b_t^{FRM} be the balance after t periods on a fixed-rate mortgage. The balance evolves according to:

$$b_t^{FRM} = (1+r)\, b_{t-1}^{FRM} - m_{t-1}^{FRM}$$

where the payment, m_t^{FRM} is given by:

$$m_t^{FRM} = \frac{r\,(1+r)^{T-t}}{(1+r)^{T-t} - 1} b_t^{FRM}$$

It is easy to show that under these definitions, the mortgage is fully amortized over T periods, and moreover that m_t^{FRM} is constant.

Recall the notation for mortgages in Section 2. We had $z_t = (a_t, r_t, b_t)$ where a_t is the age of the mortgage, r_t is the current interest rate, and b_t is the current balance. Recall also that $pay_h^m(z_t, s_t)$ is the required payment on the mortgage, and $z_{t+1} = \zeta_h^m(z_t, s_t, s_{t+1})$ controls how the mortgage evolves over time.

For a fixed-rate mortgage, we can write:

$$pay_h^m(z_t, s_t) = \frac{r_t\,(1+r_t)^{T-a_t}}{(1+r_t)^{T-a_t} - 1} b_t$$

and

$$\begin{aligned} a_{t+1} &= a_t + 1 \\ r_{t+1} &= r_t \\ b_{t+1} &= (1+r_t)\, b_t - pay_h^m(z_t, s_t) \end{aligned}$$

Fully Shared Appreciation Mortgage

A fully shared appreciation has a constant interest rate r and the payment each period is readjusted to fully amortize the remaining balance at interest rate r, over the remaining term-to-maturity. The main difference between a fixed-rate and a shared-appreciation mortgage is that the balance can be readjusted depending on what happens to house prices. The balance for a FSAM evolves according to:

$$
\begin{aligned}
b_t^{FSAM} &= \frac{p_t}{p_{t-1}}\left[(1+r)\,b_{t-1}^{FSAM} - m_{t-1}^{FSAM}\right] \\
m_t^{FSAM} &= \frac{r\,(1+r)^{T-t}}{(1+r)^{T-t}-1} b_t^{FSAM}
\end{aligned}
$$

It can be shown that

$$
\begin{aligned}
b_t^{FSAM} &= \prod_{j=1}^{t}\left(\frac{p_j}{p_{j-1}}\right) b_t^{FRM} \\
m_t^{FSAM} &= \prod_{j=1}^{t}\left(\frac{p_j}{p_{j-1}}\right) m_t^{FRM}
\end{aligned}
$$

which implies that the FSAM fully amortizes over T periods ($b_T^{FRM} = 0$) and that the balance and mortgage payments can go up or down depending on the evolution of house prices. It is also true that as long as the initial LTV is below 1, then future LTV will always be below 1 as well, so the borrower is never underwater.

Partially Shared Appreciation Mortgage

In contrast to the FSAM, the PSAM's balance evolves according to:

$$
\begin{aligned}
b_t^{PSAM} &= \min\left\{\frac{p_t}{p_{t-1}}, 1\right\}\left[(1+r)\,b_{t-1}^{PSAM} - m_{t-1}^{PSAM}\right] \\
m_t^{PSAM} &= \frac{r\,(1+r)^{T-t}}{(1+r)^{T-t}-1} b_t^{PSAM}
\end{aligned}
$$

which again implies that the PSAM fully amortizes over T periods. In the PSAM, the balance and mortgage payment can only go down. It is again true that if the initial LTV is below 1, then future LTV will always be below 1 as well.

B Evidence on within-market movers

In this section, I use data from the American Community Survey (ACS) to explore the extent to which my results may be affected by the assumption that homeowners who move will move to a different housing market. Within-city movers are an important subset of total movers. According to the ACS in 2005, about 50% of recent movers (households who moved within 1 year of the survey) moved from within the same metropolitan area. Therefore, I cannot justify my assumption simply by claiming that there are few households who move within a housing market. However, if within-market movers tend to move between houses of similar value, then equilibrium pricing will not be significantly affected because there is no net demand or supply being created within a segment of the housing market (broadly defined).

Using ACS data, I first show that, conditional on owning a home, life-cycle changes to housing value are small compared to initial differences in housing value due to income and education. Figure 1 plots average log house values across the U.S. in 2005 as a function of age, for college educated and non-college educated homeowners. The figure shows that although there are significant increases to housing value from age 25 to age 40, these differences are small compared to initial differences in housing value due to education. The average house value for a 25 year old college-educated homeowner is greater than the average house value of a 40 year old non-college educated homeowner. This evidence supports the mechanism in my model where the *initial* housing decision is more determinative of future housing value than changes to housing value over time. Figure 2 shows a similar plot for households above and below the median income level at each age group. Figure 2 tells much the same story as Figure 1—that initial differences in education and income play a larger role in determining house value than changes over the life-cycle.

Although differences in education and income play a large role in determining initial house value, there are still significant changes to housing value over the life cycle. The question, however, is rather these changes over the life cycle are induced by *moving* between owned homes, or whether they are induced by buying a first home at different points in the life cycle (and therefore at different levels of wealth). Unfortunately, ACS data do not allow me to distinguish between movers who are moving from a previously owned home and movers who are buying a home after having rented. The data therefore precludes a direct test of the change in housing value at the time of a move from one owned home to another. However, the ACS data do allow me to investigate whether the housing value of recent movers differs systematically from the housing value of owners who did not move. If homeowners tend to upgrade their homes significantly at the time of a move, then *ceteris paribus*, recent movers should have higher average housing values than homewoners who did not move. To investigate this, I run the following regression:

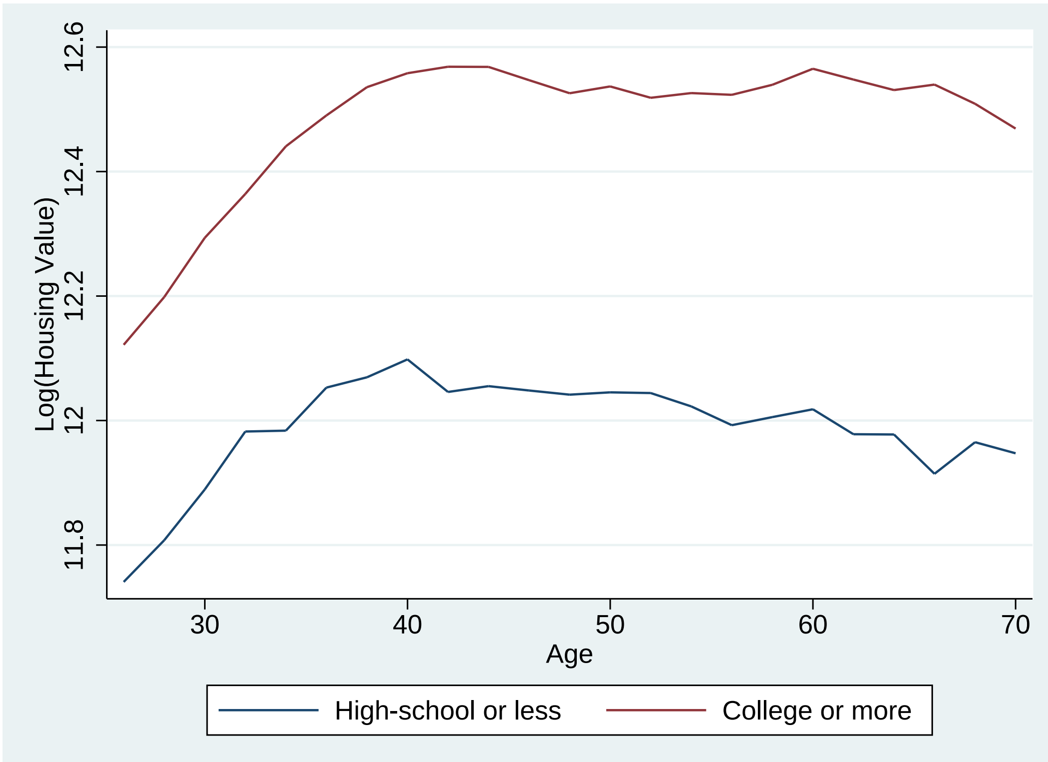
$$y_i = \beta_0 + \beta_1 InMove_i + \beta_2 OutMove_i + X_i\beta_3 + \epsilon_i \quad (1)$$

where y_i is the reported log housing value of the owner, $InMove_i$ is an indicator for whether the owner moved within the past year (from within the metro area), $OutMove_i$ is an indicator for whether the owner moved within the past year (from outside the metro area), and X_i is a set of controls including a quadratic for household income and dummies for the age of the homeowner, the year of the survey, the metropolitan area, and the race, education level, and employment status of the homeowner. The results of this regression are reported in Table 1, column 1. The results indicate that homeowners who recently moved from outside the metropolitan area tend to purchase homes of slightly higher value (1 percent) than homeowners in the metropolitan area who did not recently move. In contrast, homeowners who recently moved from within the metropolitan area do not appear to purchase homes that are different from the average homeowner in the metropolitan area. If within-market movers have an overall tendency to upgrade or downgrade when they move, there should have been a significant coefficient on $InMove_i$.

One possibility for the lack of a significant coefficient is that some homeowners upgrade and some downgrade, and that on net these two effects cancel out. To investigate this possibility, I re-run regression (1) separately from homeowners under age 45 and for homeowners over age 45. Homewoners under the age of 45 are more likely to upgrade when they move and homeowners over the age of 45 are more likely to downgrade when they move. This is confirmed in columns 2 and 3 of Table 1, which shows that recent movers under the age of 45 tend to have higher housing value than observationally similar owners who did not recently move, and vice versa for movers over the age of 45. The results imply that there is a tendency to upgrade for young homeowners and a tendency to downgrade for old homeowners. However, the magnitude of the upgrades and downgrades appear to be fairly small. On average, the housing value of a recent young mover is only 4.5 percent higher than the housing value of a similar non-mover. On average, the housing value of a recent old mover is only 5 percent lower than the housing value of a similar non-mover. These results imply that the large differences in housing value between cohorts aged between 25 and 40 are driven mostly by first-time buyers who are buying at different age and wealth levels.

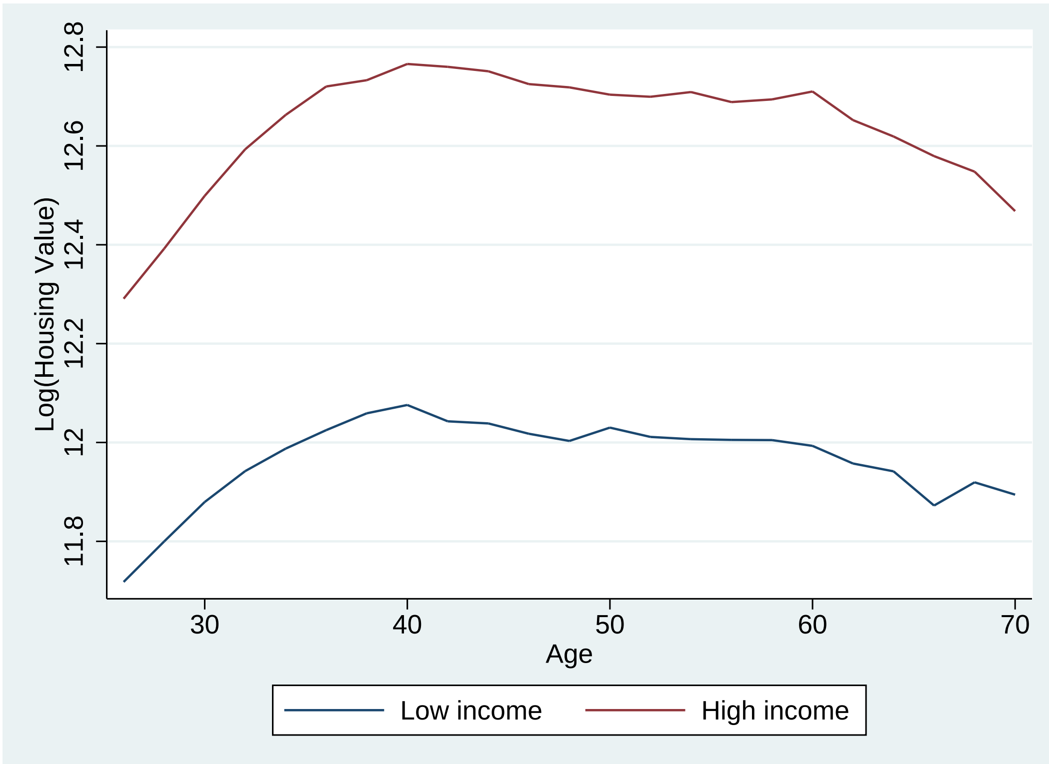
Overall, it appears that the assumption that homeowners who move move to a different housing market is a reasonable first approximation. Although the data suggests that there are within-market movers, these movers do not appear to make large changes to their housing value each time they move. Therefore, in a model with two housing segments that have large value differences, no supply is created in either segment of the housing market by within-market movers. The data suggests that differences in housing value are driven predominantly by differences initial wealth, income and education at the time of first purchase. These are mechanisms are captured by the model.

Figure 1: Differences in housing value across age cohorts and education (2005)



Note: This figure shows average log housing value for different age cohorts and education groups using U.S. ACS data from 2005. Although there are large life-cycle differences in housing value between the ages of 25 and 40, these differences are small compared to differences in initial housing value due to educational differences.

Figure 2: Differences in housing value across age cohorts and income (2005)



Note: This figure shows average log housing value for different age cohorts and income groups using U.S. ACS data from 2005. Although there are large life-cycle differences in housing value between the ages of 25 and 40, these differences are small compared to differences in initial housing value due to income differences.

Table 1: Differences in housing value between movers and stayers

	(1) All ages	(2) Age<45	(3) Age≥45
$InMove_i$	0.0047*	0.0458***	-0.0488***
	(0.0026)	(0.0032)	(0.0041)
$OutMove_i$	0.0105***	0.0561***	-0.0379***
	(0.0027)	(0.0034)	(0.0041)
N	2,439,293	685,580	1,753,713

Note: This table reports the results from regression (1) where log housing value is regressed on indicators for whether the owner recently moved, from either outside or within the metropolitan area. Controls include a quadratic in household income, and dummies for the year of the survey, the metropolitan area, and the age, employment status, education level, and race of the owner.

C Discretization of the model

Table 2 below lists the variables in the model that are discretized and the corresponding grids for each variable. Of special note is the way in which loan amounts are discretized. First, a distinction is made between "original loan amount" and "initial amount to borrow". Because all mortgages in the baseline model are fixed-rate mortgages, the exact current balance can be computed from the original loan amount. The original loan amount is therefore the state variable which is kept track of, rather than current loan amount. Keeping track of the original loan amount allows for a more robust calculation of the exact remaining balance in each period and state-of-the-world.

In order to reduce the size of the decision space, borrowers do not freely choose their initial balance from the entire grid of original loan amounts. Instead, their borrowing is limited to 10 percent increments of the current house price. So, a borrower may choose to borrow 10%, 20%, ..., 80% of the price of the home at the time of origination when using an agency loan, and additionally may choose 90% or 100% when using a non-agency loan.

Table 2: Discretization of the Model

Variable	Description	Grid	# Grid Points
rfr_t	risk-free rate	$\{0.015,\ 0.025\}$	2
μ_t^y	mean buyer income	$\log 0.055$	1
cll_t	conforming loan limit	$\{0.4,\ 0.45,\ 0.75\}$	3
mps_t	availability of non-agency loans	$\{0,\ 1\}$	2
$\bar{v}_t$	unobserved demand shock	$\left\{0.2+\left(\frac{0.7-0.2}{17}\right)n\right\}_{n=0}^{n=17}$	18
y_i	buyer income	$\{0.08,\ 0.15\}$	2
w_i	buyer initial wealth	$\{0.025n\}_{n=0}^{n=40}$	41
w_{it}	homeowner savings	$\{0.02\}$	1
r_t	mortgage interest rate (agency)	$\{0.03+0.005n\}_{n=0}^{5}$	5
r_t	mortgage interest rate (non-agency)	$\{0.045+0.0025n\}_{n=0}^{15}$	16
r_t	mortgage interest rate (SAM)	$\{0.01+0.0025n\}_{n=0}^{32}$	33
b_t	original loan amount	$\{0.025n\}_{n=1}^{n=40}$	10
p	house price	$\{0.025n\}_{n=1}^{n=40}$	40
b'	initial amount to borrow	$\{0.1n \times p_h\left(s_t\right)\}_{n=1}^{n=10}$	10